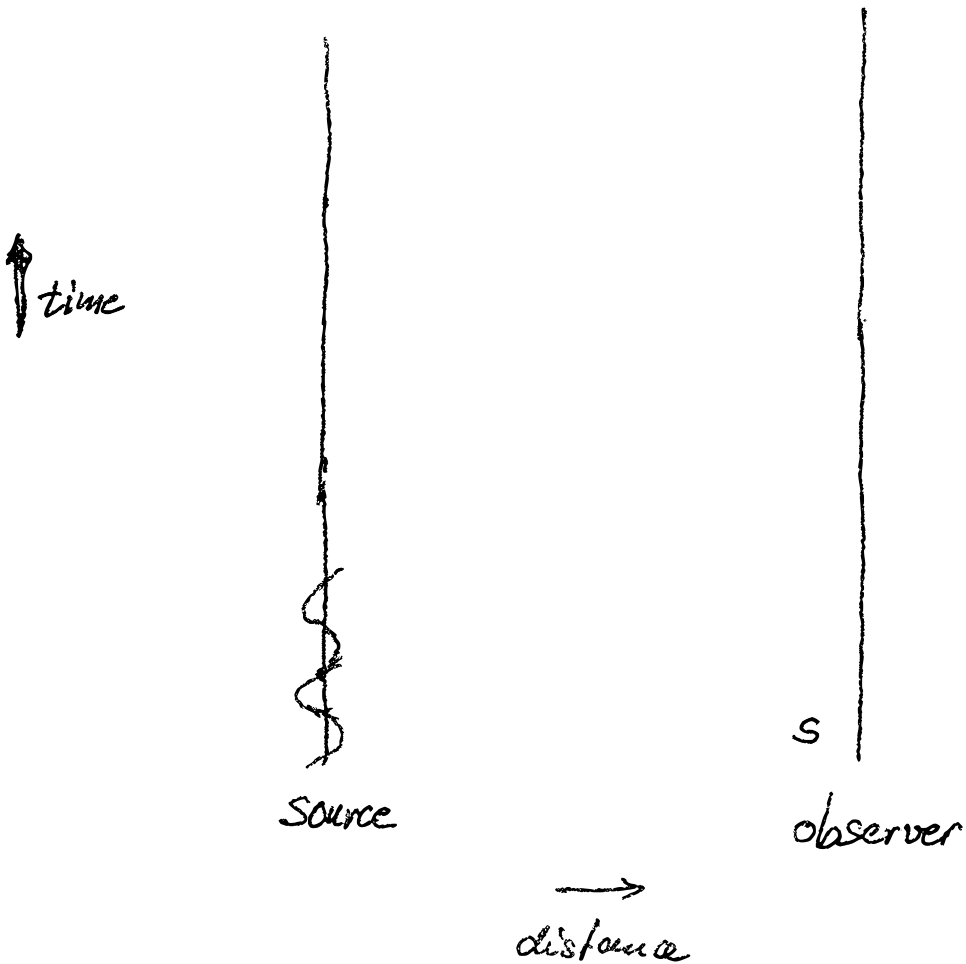
On the Bondi-Sachs mass-loss formula in non-Bondi systems

Jörg Frauendiener
University of Otago

Chris Stevens
University of Canterbury

1. The idea of null-infinity



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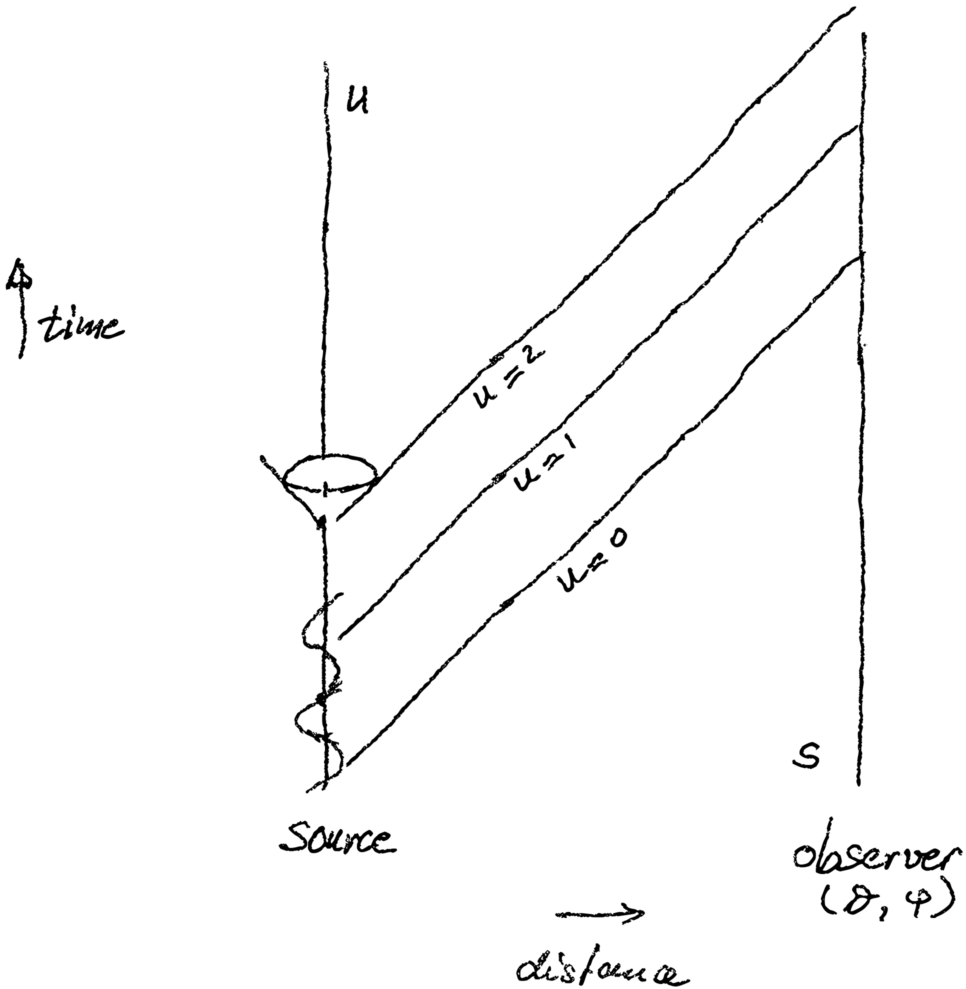

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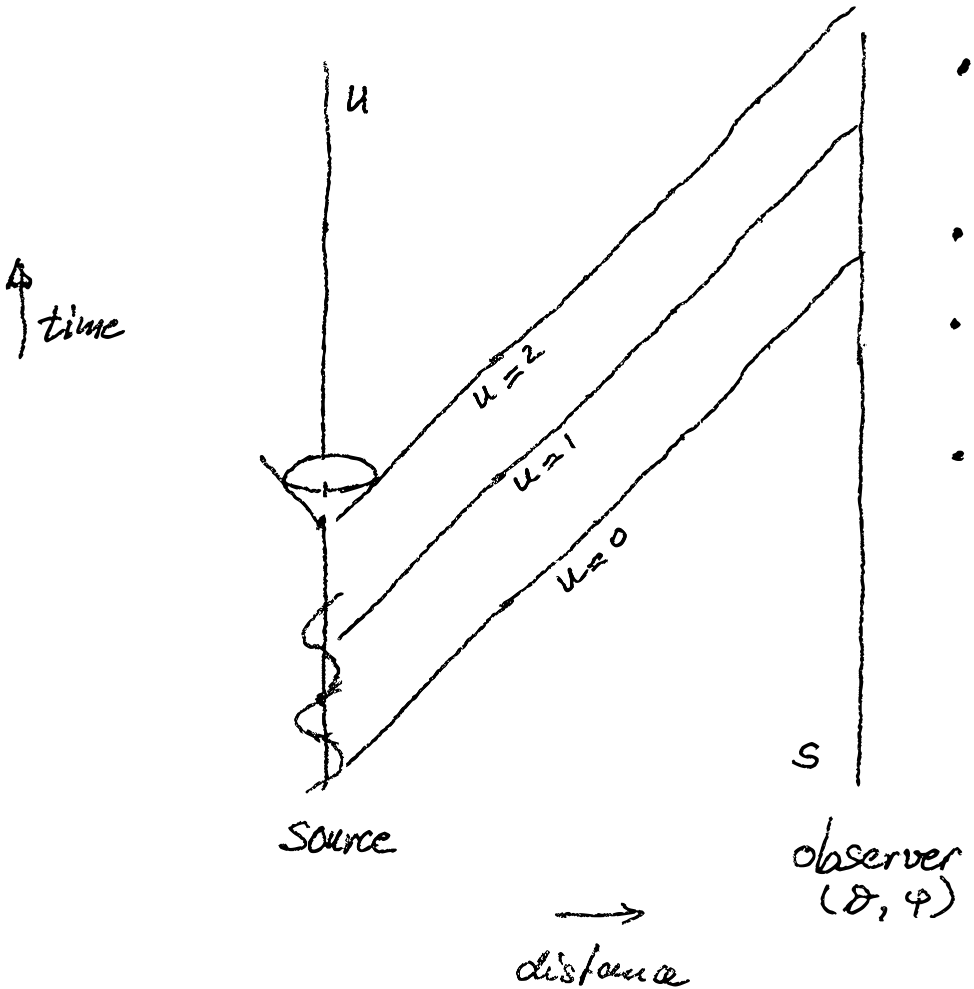
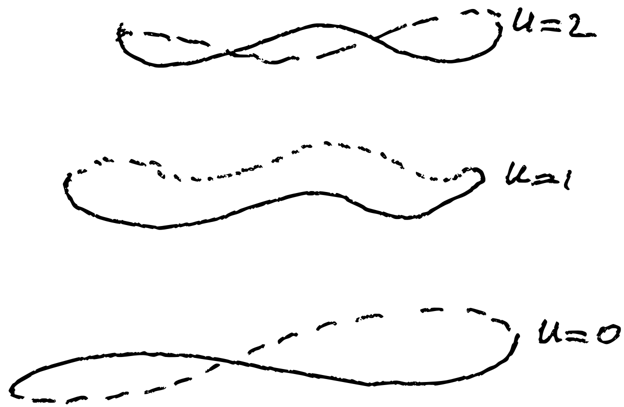
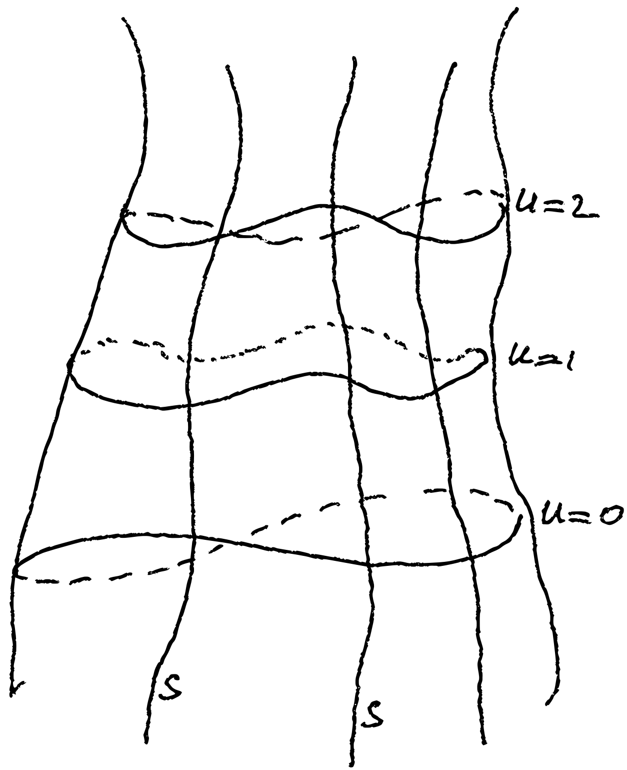


- expanding spherical wave fronts
- null hypersurface
- register at observer far away
- ideally at infinity

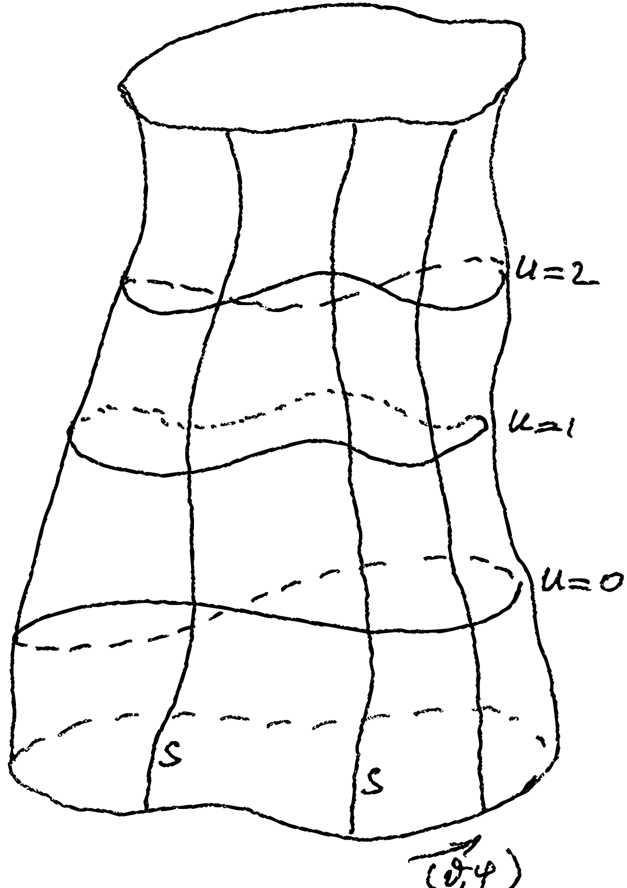
- spheres $u = \text{const}$ at $r \to \infty$ (ideal endpoints)





- spheres $u = \text{const}$ at $r \to \infty$ (ideal endpoints)

- observers at $r \to \infty$ with times s



- spheres $u = \text{const}$ at $r \to \infty$ (ideal endpoints)

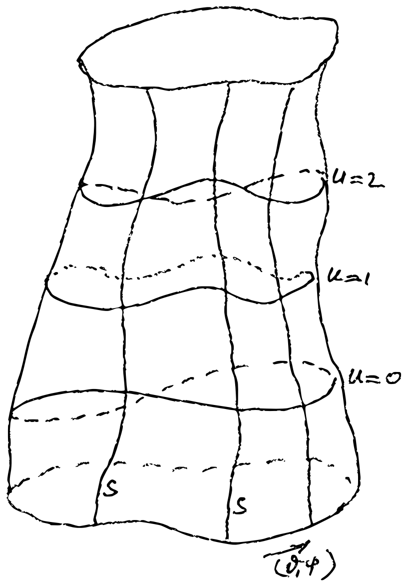
- observers at $r \to \infty$ with times s

- null infinity

  ideal endpoints of rays in all directions for all u

- null hypersurface in a related conformal spacetime

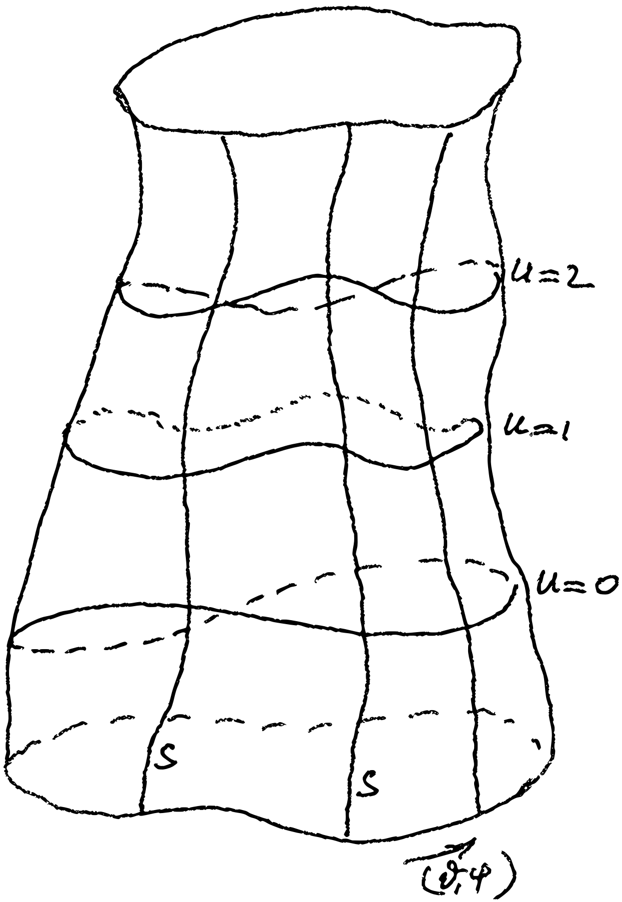
Bondi mass, news, massloss



Relevant quantities:

- wave front geometry
  expansion S'
- difference in time scales
  $\eth\left(\frac{\partial s}{\partial u}\right) = \tau$
- gravitational field
  $\Psi_0, \Psi_1, \underbrace{\Psi_2, \Psi_3, \Psi_4}_{\text{outgoing}} \quad (\sigma)$

Usually simplifying assumption
(gauge condition)



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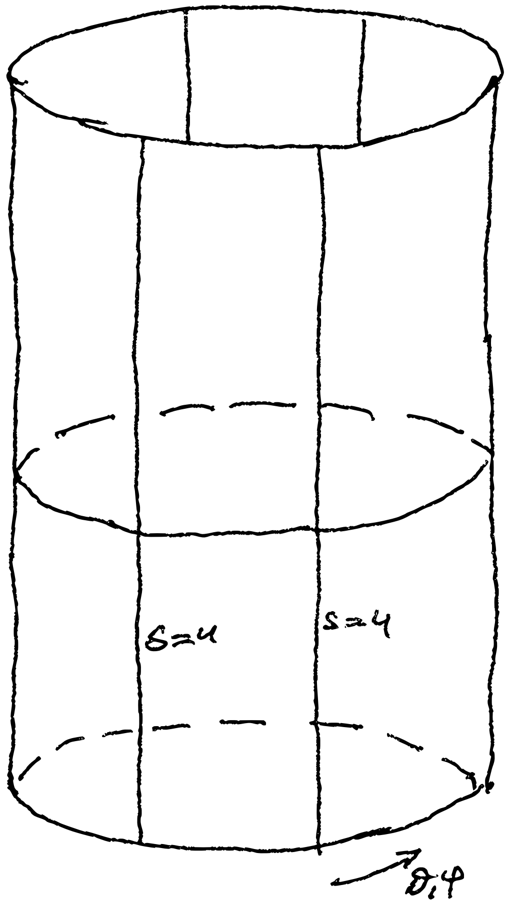

- asymptotic wave fronts have the same size (unit spheres) $\rightarrow$ no expansion $\rho' = 0$

- observer clocks are synchronised $\tau = 0$

ρ', τ NP spin coefficients

mass aspect: $\quad M = \sigma N - \Psi_2 \quad (= M(u,\vartheta,\varphi))$

news: $\quad N = -\dot{\bar{\sigma}} \quad$ (flux)

Bondi energy: $\quad m_B(u) = \int_{S_u} W_0 \, M(u,\vartheta,\varphi) \sin^2\vartheta \, d\vartheta \, d\varphi$

Bondi momentum: $\quad p_i(u) = \int_{S_u} W_i \, M(u,\vartheta,\varphi) \sin^2\vartheta \, d\vartheta \, d\varphi$

W_a "asymptotic translations" $\left(Y_{00}, Y_{1\pm1}, Y_{10}\right)$

mass-loss formula

$$m_B(u_2) - m_B(u_1) = -\int_{u_1}^{u_2}\int_{S_u} N \bar{N} \sin^2\vartheta \, du \, d\vartheta \, d\varphi \leq 0$$

$u_2 > u_1$

What about the general case?

- Wave fronts have different sizes
- Observers are not synchronized
- Occurs in numerical setups where gauges are fixed by other criteria

Problems:

- general expressions for
  - mass aspect
  - news
- space of asymptotic translations?
- inner product on that space
  (necessary for interpretation as 4-vector)
- general form of mass-loss formula

Solution:

Guiding principle : conformal invariance

- introduce conformally invariant derivatives
- generalise expressions to become conf. invariant
- define asymptotic translations as
  - solutions of conformally invariant eq'ns
  - 4-dim linear space
  - conformally invariant construction of Minkowski inner product

General formula

- news is nonlocally determined from

$$\partial_c N = A\Psi_3 \,, \quad \rho_c' N = A\Psi_4 \qquad \text{(integrable)}$$

- mass aspect

$$M = A^{-1}(\sigma N - A\Psi_2)$$

- asymptotic translation solves

$$\partial_c^2 U = R\, U \quad \text{with} \quad \partial_c R = \partial_c' Q$$

co-curvature

"Gauss" curvature

- Bondi energy momentum

$$m[U] = \int_S U \, M \, d^2\omega$$

$U > 0$: energy

see JF and C. Stevens, CQG 39, 025007 (2022)

General formula

- Bondi mass-loss formula (energy and momentum)

(i) $m_2[U] - m_1[U] = -\frac{1}{4\pi}\int_1^2 U\, N\bar{N}\, d^3V$

(ii) if $U > 0$ (time translation) then

$$m_2[U] > m_1[U]$$

(mass-loss due to energy flux carried away)

- essentially equivalent to Geroch (1977) but better adapted to our needs

see JF and C. Stevens, CQG **39**, 025007 (2022)