Tidal effect on the gyroscopic precession around a compact star

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Introduction

- The tidal effect is important in understanding the spacetime around objects of all astrophysical scales.
- The tidal deformation is one of the few features which distinguishes binaries involving BH and the ones involving only extended objects like compact stars.
- We study how to model the tidal effect on a neutron star.
- Then we study how the tidal effect will affect the overall precession frequency of gyro (GPF) orbiting the neutron star.

Figure 1: (a)Immense tidal deformation of the neutron stars in binary just before merging.(b)Two black holes in binary.

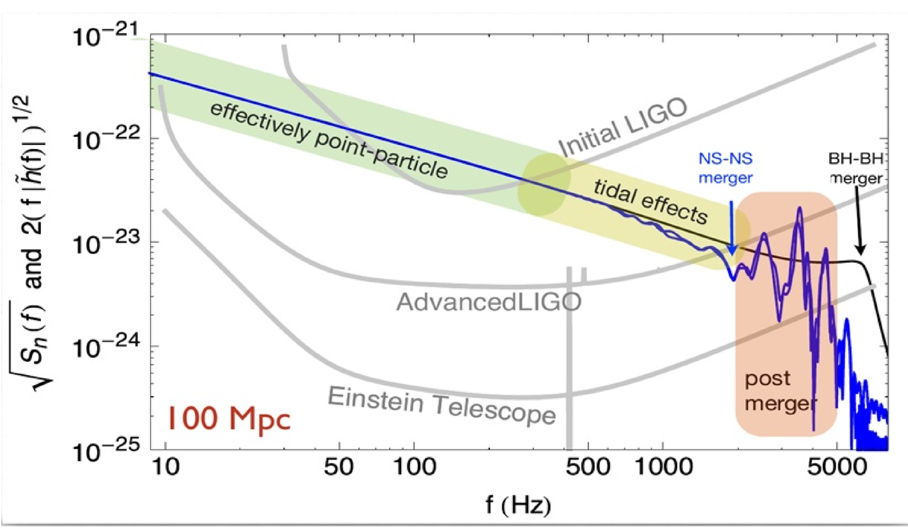

Figure 2: Sensitivity curve and signal to noise ratio for the graviational waves from binary NS merger as well as binary black hole merger.

Tidal deformation in general relativity

- In the HS's (Host star) local asymptotic rest frame, the metric coefficient g_{tt} at large r is given by:

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O\left(r^3\right), \tag{1}$$

- In the Newtonian limit, Q_{ij} is related to the density perturbation $\delta\rho$ as

$$Q_{ij} = \int d^3x\delta\rho(\mathbf{x})\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right) \tag{2}$$

and $\mathcal{E}_{ij}$ is given in terms of the external gravitational potential $\Phi_{\rm ext}$ as

$$\mathcal{E}_{ij} = \frac{\partial^2\Phi_{\rm ext}}{\partial x^i \partial x^j}. \tag{3}$$

- The induced quadrupole takes the form:- $Q_m = -\lambda E_m$ and the tidal perturbation is given by:

$$g^T_{\mu\nu} = diag\Big[-e^{\nu(r)}H_0(r), e^{\mu(r)}H_2(r), r^2K(r), r^2\sin^2\theta K(r)\Big]Y_{2m}(\theta,\phi) \tag{4}$$

- The metric describing the rotating NS with angular velocity Ω is given by

$$g^{\Omega}_{\mu\nu} = -e^{\nu(r)}[1+2(h_0(r)+h_2(r)P_2(\cos\theta))]dt^2 + e^{\mu(r)}\Big[1+\frac{2e^{\mu(r)}}{r}(m_0(r)+m_2(r)P_2(\cos\theta))\Big]dr^2 + r^2[1+2k_2(r)P_2(\cos\theta)]\Big(d\theta^2+\sin^2\theta(d\phi-\omega(r)dt)^2\Big) \tag{5}$$

where $\omega(r)$ is the frame-dragging frequency and h_0, m_0, h_2, m_2, k_2 are perturbative correction up to quadrupole.

- The tidally deformed metric for a rotating star is given by

$$g_{\mu\nu} = g^S_{\mu\nu} + g^T_{\mu\nu} + g^{\Omega}_{\mu\nu}. \tag{6}$$

where $g^S_{\mu\nu}$ is the metric for static spacetime.

- The total frame-dragging is defined as:

$$\bar{W} \equiv \omega + W. \tag{7}$$

where W is the tidal correction to frame-dragging term.

- Using various symmetries the $g_{t\phi}$ component of Einstein's equation reduces to a coupled frame-dragging differential equation for $\bar{W}$ which is given by

$$\frac{d^2\bar{W}}{dr^2} + F_1(r)\frac{d\bar{W}}{dr} + F_2(r)\bar{W} = F_3(r,\theta)\Big[S_1(r,\theta) + S_2(r,\theta) + S_3(r,\theta) + S_4(r,\theta) - S_5(r,\theta) + S_6(r,\theta)\Big] \tag{8}$$

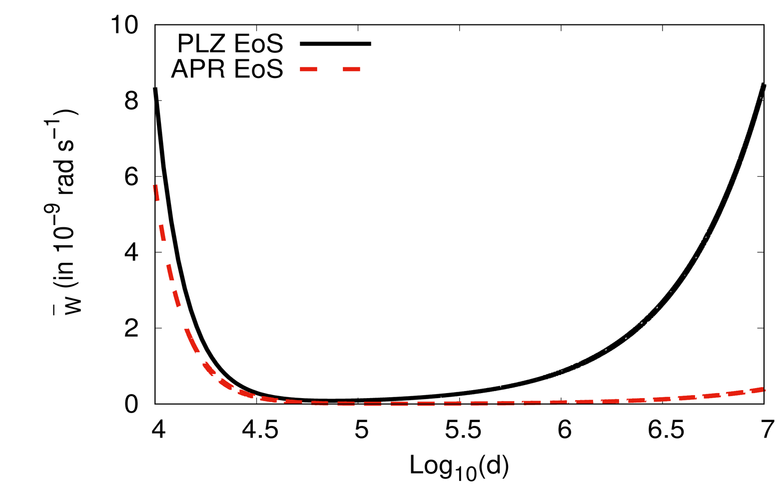

Figure 3: The variation of $\bar{W}$ with $Log_{10}(d)$ (d is in km) is shown in the figure. The separation between the HS and CS (Companion star) is ~ 2 AU.

Calculation of Overall Precession Frequency

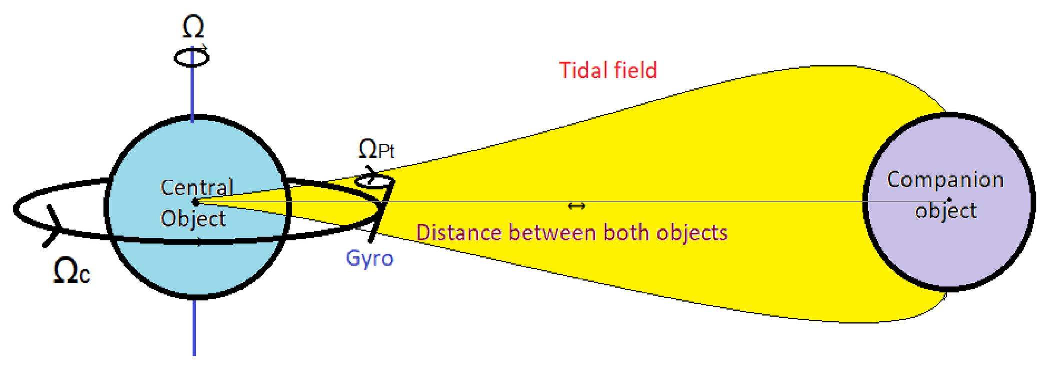

Figure 4: The sketch shows the qualitative strength of the tidal field caused by CS and it's effect on the central object or HS. Here the HS is a rotating NS or a WD.

- In a stationary and axisymmetric ST with coordinates t, r, θ, ϕ, the GPF ($\vec{\Omega}_P$) becomes:

$$\vec{\Omega}_P = \frac{1}{2\sqrt{-g}\left(1+2\Omega_c\frac{g_{t\phi}}{g_{tt}}+\Omega_c{}^2\frac{g_{\phi\phi}}{g_{tt}}\right)}.\Big[-\sqrt{g_{rr}}\Big[\left(g_{t\phi,\theta}-\frac{g_{t\phi}}{g_{tt}}g_{tt,\theta}\right)+\Omega_c\left(g_{\phi\phi,\theta}-\frac{g_{\phi\phi}}{g_{tt}}g_{tt,\theta}\right) + \Omega_c{}^2\left(\frac{g_{t\phi}}{g_{tt}}g_{\phi\phi,\theta}-\frac{g_{\phi\phi}}{g_{tt}}g_{t\phi,\theta}\right)\Big]\hat{r} + \sqrt{g_{\theta\theta}}\Big[\left(g_{t\phi,r}-\frac{g_{t\phi}}{g_{tt}}g_{tt,r}\right)+\Omega_c\left(g_{\phi\phi,r}-\frac{g_{\phi\phi}}{g_{tt}}g_{tt,r}\right) + \Omega_c{}^2\left(\frac{g_{t\phi}}{g_{tt}}g_{\phi\phi,r}-\frac{g_{\phi\phi}}{g_{tt}}g_{t\phi,r}\right)\Big]\hat{\theta}\Big]. \tag{9}$$

Where Ω_c is the orbital angular frequency of gyro revolving in a circular orbit, the magnitude of GPF is denoted by Ω_P.

Graphical results

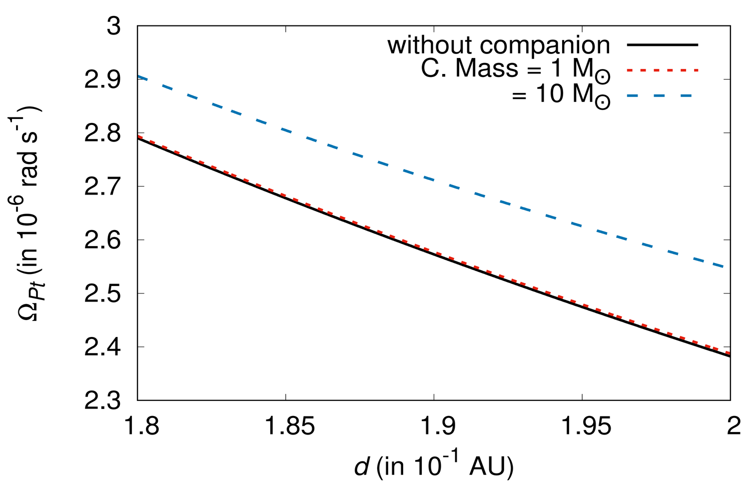

Figure 5: The variation in GPF with d is shown. The mass of CS is taken as 1 $M_\odot$ and 10 $M_\odot$.

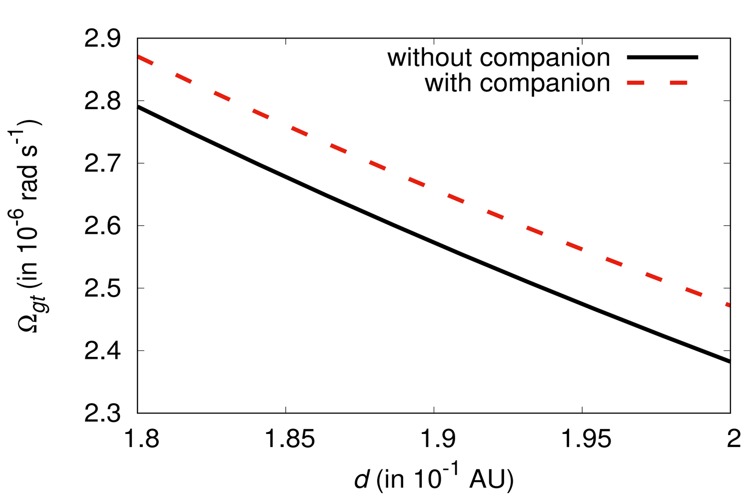

Figure 6: Plot showing the change of orbital angular velocity of geodesic with d.

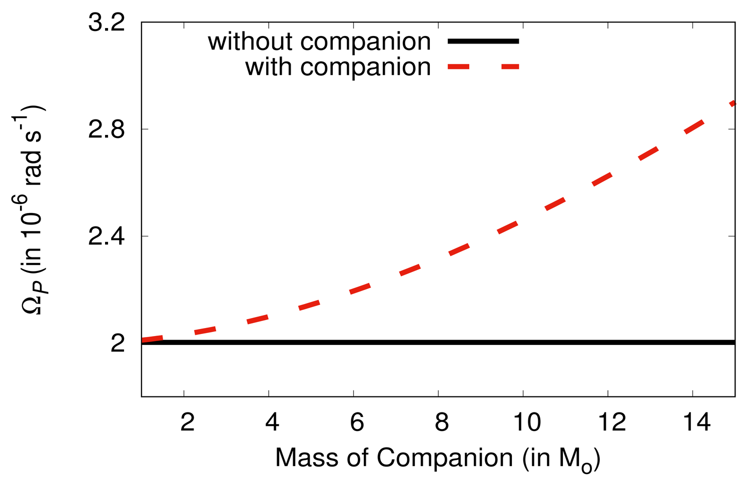

Figure 7: The figures shows the change in GPF with increasing mass of CS. The gyro is kept at a distance of $\sim$0.2 AU from the HS centre. Separation between the stars is $\sim$2 AU.

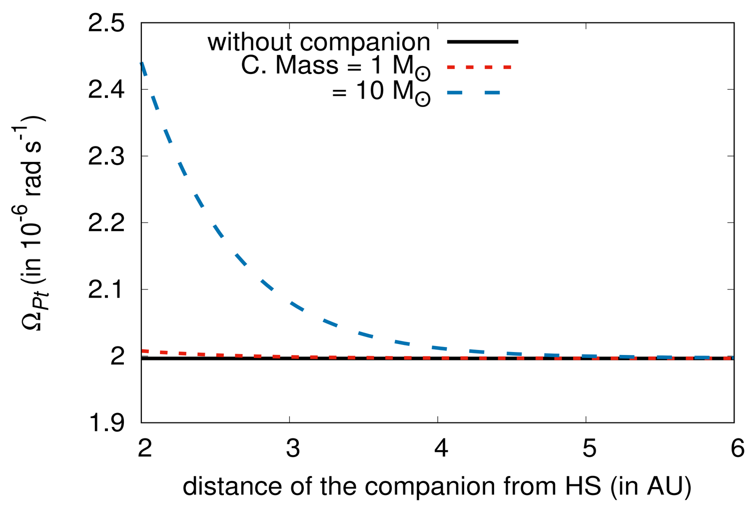

Figure 8: Plot showing the variation of GPF with increasing separation between HS and CS (2-6 AU). The gyro is kept at a distance of 0.2 AU from the HS.

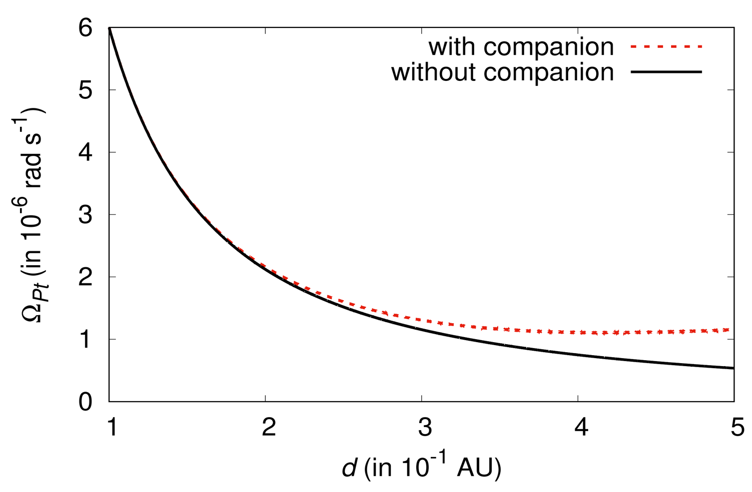

Figure 9: The variation in GPF with d with and without CS is shown. The central object is a White Dwarf.

Importance of gyroscopic precession

- As a planetary system can act as gyro, the nature of the central object can be identified.
- It provides a qualitative identification of the companion based on its mass by measuring the amount of change in precession frequency.
- The precession frequency of particles in accretion disk around the final object will rise to very high value in case of BH unlike the case of NS where it is always finite.

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