Dynamics of scalar hair with self-interaction around Schwarzchild black hole[a]

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1. Introduction

In general relativity, the no-hair theorems [1] proved that minimally coupled, potentially self-interacting scalar fields all have the trivial configurations around the stationary and asymptotically flat black holes. However, when the nonminimally coupling between the scalar field and Gauss-Bonnet (GB) term is considered [2], the no-hair theorems are violated and the nontrivial scalar configurations can form around black holes. In this paper, we study the dynamics of scalar hair around a Schwarzchild black hole in the scalar GB gravity. We work perturbatively in the coupling constant and ignore the back-reaction of the scalar hair and the GB invariant. We evolve the scalar field in the background of a Schwarzchild black hole and study the dynamical formation of scalar hair with different self-interactions.

2. Setup

The action is [3]

$$S=\int d^4x\sqrt{-g}\Big\{\frac{R}{16\pi G}+\mu\Big[-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-V(\phi)+\eta f(\phi)\mathcal{G}\Big]\Big\}, \tag{1}$$

where the GB invariant $\mathcal{G}=R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}-4R^{\mu\nu}R_{\mu\nu}+R^2$, the potential $V(\phi)=\frac{1}{2}m_\phi^2\phi^2+\frac{1}{2}\lambda\phi^4$, $f(\phi)=\phi$. The fields equations are

$$G_{\mu\nu}+16\pi\mu\eta\mathcal{G}^{GB}_{\mu\nu}=8\pi\mu T^{(\phi)}_{\mu\nu},\ \ \nabla_\mu\nabla^\mu\phi-\frac{\partial V(\phi)}{\partial\phi}=\eta\mathcal{G}. \tag{2}$$

Decomposing the $4D$ spacetime into $(\sum_t,\gamma_{ij})$ with $3+1$ decomposition: $ds^2=-\alpha^2dt^2+\gamma_{ij}(\beta^idt+dx^i)(\beta^jdt+dx^j)$. The stable evolution of spacetime is realized by the BSSN forms [4]. Using the conjugate momentum $\Pi=-\frac{1}{\alpha}(\partial_t\phi-\beta^i\partial_i\phi)$ of the scalar field, the equation of motion for the scalar field becomes

$$\partial_t\phi=\beta^i\partial_i\phi-\alpha\Pi,\ \ \partial_t\Pi=\beta^k\partial_k\Pi-\alpha D^iD_i\phi-\gamma^{ij}D_i\alpha D_j\phi+\alpha K\Pi+\alpha\frac{\partial V(\phi)}{\partial\phi}+\alpha\eta\mathcal{G}. \tag{3}$$

3. Models and physics extraction

We adopt the initial data $\phi_0=\Pi_0=0$ and evolve the system by using Einstein Toolkit with Maya code [5].

$\bar{m}_\phi$ \ λ	0.0	10	100	1000
0.0	sf11	sf12	sf13	sf14
0.5	sf21	sf22	sf23	sf24
1.0	sf31	sf32	sf33	sf34
1.5	sf41	sf42	sf43	sf44

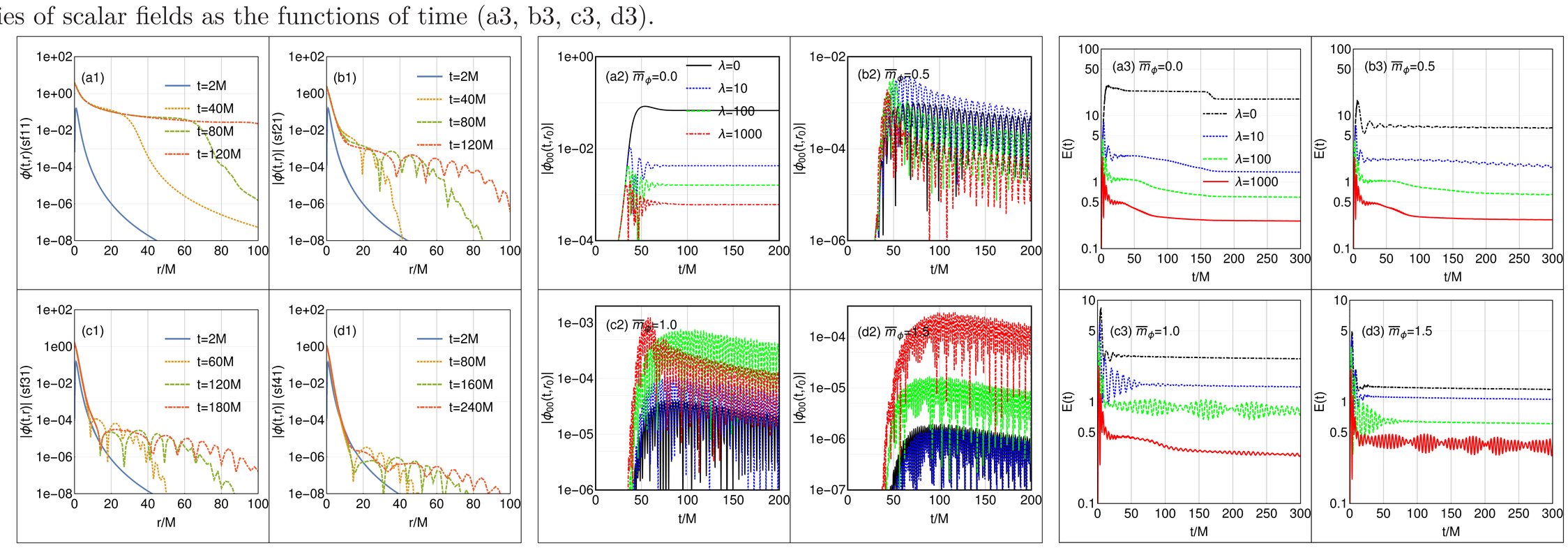
$$\phi_{lm}(t,r) = \int d\Omega\phi(t,r,\theta,\varphi)Y^*_{lm}(\theta,\varphi), \tag{4}$$

$$E_{rad}(t) = \int_0^t\Big(\lim_{r\to r_c}r^2\oint T_{tr}\,d\Omega\Big)dt', \tag{5}$$

$$E_{sf} = \int\alpha\sqrt{\gamma}\,T^{(\phi)}_{\mu\nu}n^\mu n^\nu\,d^3x. \tag{6}$$

4. Numerical results

We give the profiles of scalar fields at different instances (a1, b1, c1, d1), waveforms of scalar fields measured at $r=30M$ (a2, b2, c2, d2), and the total energies of scalar fields as the functions of time (a3, b3, c3, d3).



5. Conclusions and Acknowledgments

In this paper, we investigated the dynamical formation of scalar hair with the self-interaction in the scalar GB gravity. We obtained the configurations of the scalar field at different instances and found that all the scalar fields will relax to static configurations at late time. For the massless scalar field, we found that the radiated energy decreases with the coupling parameter λ. For the massive cases, the radiated energies will increase or decrease with the coupling parameter λ, and it depends on the mass parameter m_ϕ. We found that the introduction of the self-interaction reduces the total energy of the scalar hairs at late time, and the corresponding total energy decreases with the mass parameter m and the coupling parameter λ.

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6. References

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