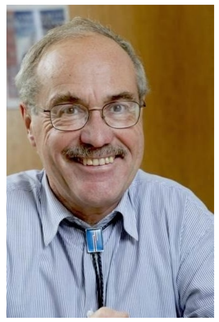

The Scale Invariant Vacuum Paradigm

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Objectives

Review of the key results and current progress of the Scale Invariant Vacuum (SIV) idea within the Weyl Integrable Geometry (WIG) paradigm:

- Comparing $a(t)$ within ΛCDM and SIV.
- Scale-Invariant Dynamics of Galaxies.
- SIV & the growth of the density fluctuations .
- SIV and the Inflation of the Early Universe.

Motivation

Scale Invariance and Physical Reality

- The laws of physics change upon change of scale! Consistent units is paramount for dimensional estimates. Matter usually defines a relevant scale.
- However, an empty Universe is scale invariant! Maxwell equations are scale invariant in vacuum. The field equations of GR are scale invariant for empty space with no cosmological constant.
- At what scale and in which systems scale invariance is possible and actually manifested?

Einstein GR & Weyl Integrable Geometry.

- In GR metric-compatible connection guarantees $\delta\|\vec{v}\| = 0$ along a geodesic.
- Could the Dark Matter and Dark Energy phenomena be artifacts of deviations $\delta\|\vec{v}\| \approx 0$ that accumulate over cosmic distances?
- In Weyl Integrable Geometry $\oint \delta\|\vec{v}\| = 0$ along a closed loop - this defeats the Einstein objection!

Backgrownd

WIG and Dirac co-calculus [1, 2]:

- Weyl Geometry uses $g_{\mu\nu}$, k_μ, and a scalar field λ.
- Covariant derivatives based on Dirac co-calculus.
- In the Weyl Integrable Geometry $k_\mu = -\partial_\mu \ln(\lambda)$.

Gauge change and derivatives:

Einstein GR objects have prime "$\prime$" otherwise WIG.

- $l' \to \lambda(x) l \Leftrightarrow ds' = \lambda ds \Rightarrow g'_{\mu\nu} = \lambda^2 g_{\mu\nu}$,
- co-tensor of power n: $Y'_{\mu\nu} \to \lambda^n Y_{\mu\nu}$,
- co-scalar of power n: $S_{*\mu} = \partial_\mu S - n k_\mu S$,
- for co-vector of power n: $A_{\nu *\mu} = \partial_\mu A_\nu - n k_\nu A_\mu - {}^*\Gamma^\alpha_{\nu\mu} A_\alpha$,
- for co-co-vector of power n: $A^\nu_{*\mu} = \partial_\mu A^\nu - n k^\nu A_\mu + {}^*\Gamma^\nu_{\mu\alpha} A^\alpha$,
- where ${}^*\Gamma^\nu_{\mu\alpha} = \Gamma^\nu_{\mu\alpha} + g_{\mu\alpha} k^\nu - g^\nu_\mu k_\alpha - g^\nu_\alpha k_\mu$.

Cosmology in Scale Invariant gauge:

Ricci tensor & scalar in Weyl and Einstein' frames, Dirac [2] and by Canuto et al. [3].

$$R = R' + 6\kappa^\alpha\kappa_\alpha - 6\kappa^\alpha_{;\alpha},$$
$$R_{\mu\nu} = R'_{\mu\nu} - (\kappa_{\mu;\nu} + \kappa_{\nu;\mu} + 2\kappa_\mu\kappa_\nu) + + g_{\mu\nu}(2\kappa^\alpha\kappa_\alpha - \kappa^\alpha_{;\alpha}).$$

Einstein Equation:

$$R_{\mu\nu} - g_{\mu\nu}R/2 = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu},$$
$$R'_{\mu\nu} - g_{\mu\nu}R'/2 - (\kappa_{\mu;\nu} + \kappa_{\nu;\mu} + 2\kappa_\mu\kappa_\nu) + + g_{\mu\nu}(2\kappa^\alpha_{;\alpha} - \kappa^\alpha\kappa_\alpha) = -8\pi G T_{\mu\nu} - \lambda^2\Lambda_E g_{\mu\nu}.$$

The FLRW equations for Scale Invariant Cosmology

FLRW equations within the WIG were first derived in 1977 by Canuto et al. [3] and in 2017 by Maeder [4].

$$\frac{8\pi G\rho}{3} = \frac{k}{a^2} + \frac{\dot a^2}{a^2} + 2\frac{\dot\lambda\,\dot a}{\lambda\, a} + \frac{\dot\lambda^2}{\lambda^2} - \frac{\Lambda_E\lambda^2}{3} \quad \text{and} \quad -8\pi G p = \frac{k}{a^2} + 2\frac{\ddot a}{a} + 2\frac{\ddot\lambda}{\lambda} + \frac{\dot a^2}{a^2} + 4\frac{\dot a\,\dot\lambda}{a\,\lambda} - \frac{\dot\lambda^2}{\lambda^2} - \Lambda_E\lambda^2. \quad (1)$$

Scale Invariant Vacuum gauge (T=0 and R'=0) Maeder [4]

The SIV vacuum $T_{\mu\nu} = 0$ and $R'_{\mu\nu} = 0$ satisfies: $\kappa_{\mu;\nu} + \kappa_{\nu;\mu} + 2\kappa_\mu\kappa_\nu - 2g_{\mu\nu}\kappa^\alpha_{;\alpha} + g_{\mu\nu}\kappa^\alpha\kappa_\alpha = \Lambda g_{\mu\nu}$, (2)

Corollary (2017) by Maeder [4]: For homogeneous and isotropic WIG-space, where $\partial_i\lambda = 0$, only $\kappa_0 = -\dot\lambda/\lambda$ and its time derivative $\dot\kappa_0 = -\kappa_0^2$ are non-zero, then from (2) one has pair of equivalent equations:

$$3\frac{\dot\lambda^2}{\lambda^2} = \Lambda \quad \text{and} \quad 2\frac{\ddot\lambda}{\lambda} - \frac{\dot\lambda^2}{\lambda^2} = \Lambda \quad \text{or} \quad \frac{\ddot\lambda}{\lambda} = 2\frac{\dot\lambda^2}{\lambda^2} \quad \text{and} \quad \frac{\ddot\lambda}{\lambda} - \frac{\dot\lambda^2}{\lambda^2} = \frac{\Lambda}{3}. \quad (3)$$

In SIV gauge (3) **the cosmological constant disappears from the FLRW equations** (1):

$$\frac{8\pi G\rho}{3} = \frac{k}{a^2} + \frac{\dot a^2}{a^2} + 2\frac{\dot a\dot\lambda}{a\lambda} \quad \text{and} \quad -8\pi G p = \frac{k}{a^2} + 2\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + 4\frac{\dot a\dot\lambda}{a\lambda}. \quad (4)$$

Comparing ΛCDM and SIV [4, 5]

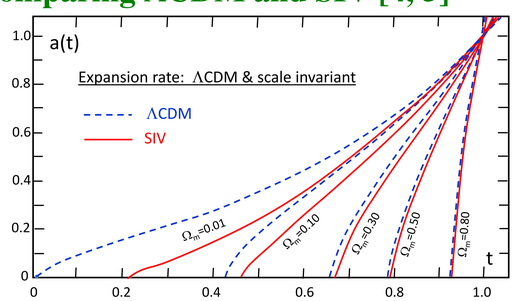

Figure 1: Expansion rates $a(t)$ as a function of time t in the flat ($k = 0$) ΛCDM and SIV models in the matter dominated era. The curves are labeled by the values of Ω_m.

Growth of the density fluctuations [6]

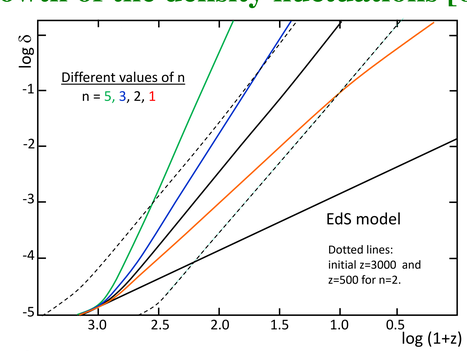

Figure 2: Initial value $\delta = 10^{-5}$ at $z = 1376$ and $\Omega_m = 0.10$. The two light broken curves show models with initial $(z+1) = 3000$ and 500, with same $\Omega_m = 0.10$ and $n = 2$.

Dwarf Spheroidals, SIV, and MOND [7]

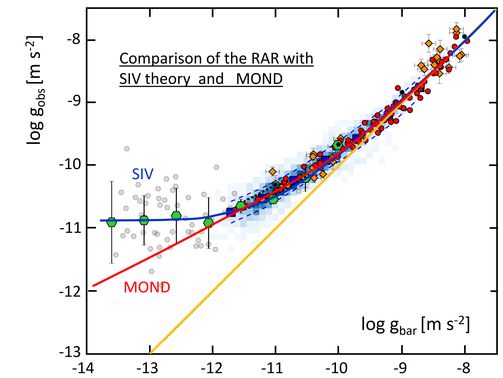

Figure 3: Radial Acceleration Relation (RAR) of g_{obs} and g_{bar} for the galaxies studied by Lelli et al. (2017). The blue curve shows the relation predicted by the SIV [7]. The red curve gives the MOND relation, while the orange shows the 1:1-line.

Key SIV Results:

- **The SIV is a viable alternative to ΛCDM.**
  - In SIV gauge (3) the cosmological constant disappears.
  - Diminishing differences at big densities (Fig. 1) [4, 5].
  - SIV has fast growth of the density fluctuations (Fig.2) [6].
  - Correct RAR for dwarf spheroidals (Fig.3) [7].
- **Early inflation is also natural** within the SIV cosmology with a graceful exit from inflation [8].

References

[1] Weyl, H. 1923, Raum, Zeit, Materie. Vorlesungen über allgemeine Relativitätstheorie. (Springer Verlag (1970)).

[2] Dirac, P. A. M., Proc. R. Soc. Lond. A, **333**, 403 (1973).

[3] Canuto, V., Adams, P. J., Hsieh, S.-H., & Tsiang, E., 1977, Phys. Rev. D **16**, 1643 (1977).

[4] Maeder, A. 2017, *An Alternative to the LambdaCDM Model: the case of scale invariance,* Astrophys. J. , **834**, 194 (2017). e-Print: 1701.03964 [astro-ph.CO].

[5] Maeder, A.; Gueorguiev, V.G., *The Scale-Invariant Vacuum (SIV) Theory: A Possible Origin of Dark Matter and Dark Energy,* Universe **6**, 46 (2020). DOI: 10.3390/universe6030046.

[6] Maeder, A., Gueorguiev, V., G., *The growth of the density fluctuations in the scale-invariant vacuum theory,* Phys. Dark Univ. **25**, (2019) 100315. e-Print: 1811.03495

[7] Maeder, A.; Gueorguiev, V.G. *Scale-invariant dynamics of galaxies, MOND, dark matter, and the dwarf spheroidals,* MNRAS **492**, 2698 (2019). e-Print: 2001.04978 [gr-qc]

[8] Maeder, A., Gueorguiev, V. G., *Scale invariance, horizons, and inflation.* MNRAS **504**, 4005 (2021). e-Print: 2104.09314 [gr-qc].