Vector particles scattering by Schwarzschild Black Holes

Alex Gaina

Institute of Applied Physics (retired)
Academiei 5 Chisinau, MD- 2028, Republic of Moldova

E- mail: alex_gaina@yahoo.com

Introduction

The vector particles quasibound states were of recent interest [12]. The authors of the cited article given a very comprehensive and almost exhaustive analysis of the problems involving vector (Proca) mass particles quasi bound states, including the case of a small rotation of a Black Hole (a<<M), when the quasibound state timelifes (dampings) and energies should vary very small from the timelifes (dampings) and energies in a Schwarzschild field. Let us mention , that the vector mass particles timelifes around Schwarzschild Black holes were investigated for a first time in the works [8, 9]. Later this investigation was repeated in the Conference article [10], which given a various result and this fact, perhaps, as well as the obscurity of the Conference on General Relativity and Gravitation held in 1988 in Yerevan influenced a series of authors from the West (see, e.g. [11]) to compute again from anew the problem as a whole.
Unfortunately, no a definite and clear results were obtained, as can be seen from ours analysis given bellow, so that the problem remained still unclear till the article [12] appeared. The formula presented in [12] give us the hope, that a qualitatively correct analysis has finally been given, with taking account of fine and hyperfine interactions between the spin of the test vector particle and the gravitational field of an almost nonrotating Black hole (nearly Schwarzschild Black hole) , as well as of the spin- spin interaction between a slowly rotating Kerr Black hole and the spin of a vector particle. This 2 new interactions put into discussion and consideration in the article [12] for a first time, allow us to hope that the most complicated case of a Kerr spacetime and vector mass particles would be finally solved successfully.
The purpose of this short poster, is to show to researchers of the problem a completely another way of treatment of the problem. Namely: Let us examine from the beginning not the bound (quasi bound , when the damping is small) problem, but the scattering problem, when the motion of a particle is infinite, or E > mc^2. In this case , which is historical one in the framework of quantum mechanical research in the Black holes backgrounds, a lot of investigations were made concerning scalar and Dirac particles, which , to my opinion, would be redundant to cite in this short letter. They are well known. It is important, that this scattering case widely investigated for scalar (Klein – Gordon) equation and Dirac equation easily lead to the finite motion case motion , if apply the scattering matrix and calculate the poles of the scattering matrix. This has been shown for a first time in the article [13] . Or , it has been shown that the problems of infinite motion of mass particles in the field of Black holes is equivalent to the problem of finite motion of mass particles, because the poles of the S- matrix allow to calculate the bound states energies and the dampings (time lifes , let us recollect , that the time lifes are just the inverse of dampings) or time lifes of mass particles on bound states.
The results obtained here can be easily used for the case of slowly rotating (Kerr) black holes.

Scalar and Dirac particles scattering

In the work [1] (see, equation (2.77) at the page 70, the PDF of PhD is available through inspirehep server) has been shown, that the ratios

$$\xi \equiv \frac{\gamma}{T(\omega=\mu)} = \text{constant} = \frac{1}{2\pi}(\mu M)^2 \frac{(n+l)!}{(n-l-1)!n^{4+2l}} \qquad (1)$$

where $\gamma -$ is damping of a quasibound level with $\omega \leq \mu$, $T(\omega) -$ transmission (absorption) coefficient of a partial wave with $\omega > \mu$. I use here and I shall use bellow almost everywhere the system of units $c = \hbar = G = 1$, if the inverse is not evident. The constant ratio in the formula (1) does not depends on the spin of the particles, and this property follows from the S-matrix scattering theory in a gravitational field. A general case for the scalar mass field was examined in the paper [2], while another cases follow from the property of spin ½ field established in (1). I shall extrapolate this property bellow for mass vector particles also.
A general expression for the damping of levels for a vector (Proca) test particles field in a Schwarzschild external gravitational field was found in the [8] in a long wavelength limit, a little bit later a generalization for a slowly charged Schwarzschild Black hole has been given in [9], while another calculation for the same problem has been given in [10].

Vector particles scattering

Unfortunately, the results in the limit of Schwarzschild field in [8] and [9] varies unsignificantly. Namely, the expression in the formula on page 5 of ref. [9] varies in a denominator from the expression in a formula (15) of the ref. [9] as $\frac{1}{2l+1}$ with $\frac{1}{2l+3}$, for $s = \pm 1$ which would lied to a difference of the order of 86% in the timelife of the vector particle in a bound state with $s = \pm 1$.
But the result in the reff. [10] differs much more significantly. The dumping of the mass Proca vector field quasibound levels has been found in (4) (see also (5)) of the reff. [9] and has the following view:

$$\gamma_l^{Proca} = \qquad (2)$$

$$= \pi\left(2\sqrt{2}\mu M\right)^{4l+6} \frac{(l+n)!}{(n-l-1)!\,(2l)!\,(2l+1)!\,(2n)^{4l+6}} \left\{ \begin{matrix} 1, for\ s = 0 \\ \sqrt{\frac{[n^2-(l+1)^2](2l-1)^2(2l-2)}{(n^2-l^2)(2l+3)(2l+2)^2}}, s = \pm 1 \end{matrix} \right\}$$

Where $n = l + n_r + 1 + s$, $l -$ orbital quantum number, $n_r -$ radial quantum number, $s = 0, -1, +1$, possible spin projections on a chosen axis of symmetry, $\mu -$ mass of the Proca test field, $M -$ mass of the Black Hole, which is supposed to be a Schwarzschild one.
It is to note, that the formula (2) is losing its meaning for $n_r = 0, S = -1$ – it becomes divergent, while for $l = 1, S = \pm 1$ it gives absolutely stable orbits. To my opinion this is due to absence of the fine and hyperfine interaction terms, which would transform the formula (2) into obvious decaying one.
Let us reproduce the result in the reff. [10]. The energy of the quasilevel is

$$\omega = \mu\left(1 - \frac{\mu^2 M^2}{2(n+\sigma)^2}\right), \qquad (3)$$

Then the authors of [10] use $j = l + 1$ for the total momentum, $n = j + n_r + s$ for the principal quantum number, so that the damping is:

$$\gamma = 4(\mu M)^{4l+5}\,(2l + n + 1)!\,\frac{[(l+1+s)!(l-1+s)!]^2}{n!(l+n+s+2)^2(2l+2)!(2l+3)!} \qquad (4)$$

A discouraging feature of the results (2) and (4) is discrepancy in the leading power of the there small parameter:

$$\mu M = \frac{G\mu M}{\hbar c} \qquad (5)$$

which represents the strength of the Newtonian attraction between vector particle and the force center entering to both formula. This discrepancy by an order of magnitude would lead to rather great variations for $\mu M \ll 1$.

An alternate calculation for this case has been given in [11], where the main accent was put on the numerical calculations. The main equations, from which the damping can be derived are (64) (for odd modes, S=0), and (81) and (82). For odd modes they conclude that the previous calculations in [8] are confident only for great orbital numbers $\gg 1$, although an explicit calculation of damping for lowest states they do not give. Unfortunately, the equation (82) which was derived for even modes ($S = \pm 1$) do not lead from (81). It contains a mistake in derivation so that the authors of [11] limited themselves further to numerical calculations of the damping. Their result for total energy is (for odd (S=0) parity modes:

$$\omega \cong \mu\left(1 - \frac{(\mu M)^2}{2(l+1+n)^2}\right) \qquad (6)$$

The total energy shift is:

$$\delta\omega = A(1 - 2i\omega)\prod_{k=1}^{l}(k^2 - 1 - 4i\omega)\,, \qquad ([11], 64)$$

where

$$A = \frac{2^{2l+2}(\mu M)^{4l+5}}{(l+1+n)^{2l+4}} \cdot \frac{(2l+1+n)!}{n!} \cdot \frac{(l+1)(l-1)}{(2l-1)!![(2l+1)!]^2} \qquad (7)$$

While for even parity ($S = \pm 1$) the equation (81 of the reff. [11]) looks like:

$$4\omega(4\omega)\left(4i\sqrt{\mu^2 - \omega^2}\right)^{2l+1}\left[\frac{l!^2}{2l!(2l+1)!}\right]^2 \prod_{k=1}^{l}(k^2 + 16\omega^2) = -1 \qquad ([11], 82)$$

As I mentioned earlier, this equation does not derive from equation (81), while, the equation (81) is not allowable for small (as compared with fine structure of Newtonlike levels) dampings.

There is need to accentuate, from the beginning the calculations in [8]-[11] are analytical but approximate. The results obtained by 3 groups of authors varies significantly. If there is an agreement for odd modes between [8,9] and [11], there is no an agreement between these 2 groups and [1] for great orbital momentums. The results in [10] varies significantly in the numerical coefficient, although the leading term (μM) in [10] and [11] coincides, but does not agree with calculations in [8] and [9].

To me the explanations of these differences is hidden in the fine and hyperfine interactions, which were not taken into account and which would lead to total agreement at least between [8, 9] and [11]. From these interactions the most important is gravitational spin-orbit interaction which would modify in such a way calculations in [8, 9] and [11], that an agreement would be attained. The gravitational spin-orbit interaction would modify the formula for levels energy (3) and (6) such a manner, that the damping would be changed significantly. This would happen, because the fine structure terms would be great for lowest states $(l = 0, 1)$. I would repeat here, that a quasistationary state with $l = 0$ would be possible, since the Compton wavelength of the Proca particle is great compared with the gravitational radius of the Black Hole. Only for small Compton wavelengths s-bound states are not possible.

Then fine structure terms would contribute significantly to the levels damping, while the oscillations of the wave function in the region inside the barrier due to absorption would change the total energy of the particle.

Coming back to the main purpose of this short letter, I would choose here for estimations of the absorption coefficient the analytical results obtained in the paper [8]. This would give at least the absorption coefficient for the scattered Proca field in the nonrelativistic limit. Inversely, if the absorption coefficient would be calculated in the long wavelength limit, then the levels damping would be calculated form the poles of the S-matrix.

The transmission coefficient for a scattered Proca wave in the nonrelativistic and long wavelength limit can be found easy from the property (1) and the formula (2) of the damping:

$$T_l^{Proca} = \pi^2(\omega M)^{4l+4}\frac{2^{3l+6}}{(l)!(2l+1)}\left\{\begin{matrix} 1, & for\ s = 0 \\ \sqrt{\frac{[n^2-(l+1)^2](2l-1)^2(2l-2)}{(n^2-l^2)(2l+3)(2l+2)}}, & for\ s \pm 1 \end{matrix}\right\}, \text{ where } \omega \cong \mu \quad (8)$$

It is not difficult to estimate the absorption cross section in the long wavelength limit $\omega M \ll 1$ for vector mass field, taking into account that the main contribution is coming from the $l = 0, S = 0$ mode:

$$\sigma_{abs}^{Proca} = \frac{\pi T_{1000}}{(\omega v)^2} = \frac{16\pi^3 {R_G}^2}{v^2}\omega^2 M^2 \equiv 4\pi^2 S_H \frac{(\omega M)^2}{v^2} \quad (9)$$

where

$R_G = \frac{2GM}{c^2}$ – gravitational radius of a Black Hole, $S_H = 4\pi {R_G}^2$ – the event's horizon surface of a Black Hole, $v = \sqrt{1-\left(\frac{\mu}{\omega}\right)^2} \equiv \sqrt{1-\left(\frac{\mu c^2}{\hbar\omega}\right)^2}$.

In the limit $v \to 1$ this cross section would be by $9\pi^2$ times greater than the photonic cross section. There is also to be noted, that for $\omega M \sim 1$ the transmission coefficient and the absorption cross section, which would be created by $l \geq 1$ modes also, would have represents diffraction

phenomena. It is also normally to expect, that in the short wavelength limit the absorption cross section would exhibit the obvious ultra relativistic (light ray) $\sigma = 27\pi M^2 = \frac{27\pi {R_G}^2}{4} = \frac{27}{4} S_H$ shape.

To estimate correctly the gravitational spin- orbit term would be useful to calculate this term in a weak field approximation, as in [15] and a refference therein.

ORCID iD

Alexei Gaina iD https://orcid.org/0000-0003-955-4778

References:

[1] Gaina A B 1981 *Quantum particles in Kerr – Newman fields* (PhD work (in Russian), Moscow, Moscow State University) available through inspirehep server

[2] Gibbons G W 1975 *Commun. Math. Phys.* **44** 246

[3] Ford L H 1975 *Phys. Rev.* **D12** 2963

[4] Kharkov A A 1972 *Preprint No*. 73-72 (Novosibirsk: Institute of Nuclear Physics)

[5] Unruh W 1976 *Phys. Rev.* **D14** 3251

[6] Gaina A B 1980 *Scattering of scalar particles and electrons in a Reissner – Nordstrom field* (Moscow VINITI No. 1970-80 (in Russian)) 20 pp.

[7] Gaina A B, Chizhov G A 1983 *Moscow Univ. Phys. Bull.* Moscow **38** 1

[8] Galtsov D V, Pomerantseva GV Chizhov G A 1984 *Sov. Phys. J.* **27** 697

[9] Pomerantseva G V 1985 *Massive fields in Black Holes vicinities* (Moscow PhD Abstract Moscow State University (in Russian)) 5 pp

[10] Volkov M S, Ershov A A 1988 *Behavior of a massive vector field in a Schwarzschild spacetime* (Yerevan: Soviet Gravitational Conference, Yerevan State University (in Russian)) pp 260-261

[11] Rosa J G, Dolan S 2012 *Phys. Rev.* **D85** d4043

[12] Baumann D, Chia H S, Stout J, ter Haar L 2019 *J. Cosmology Astropart. Phys.* 12 id 006(2019) arXiv:1908.10370

[13] Gaina A B, Kochorbe F G 1987 *Sov. Phys. JETP* **65**(2) 211

[14] Starobinskii A A 1973 *Sov. Phys. JETP* **37**(1) 28

[15] Gaina AB Ionescu- Pallas NI 1997 *Rom. Repts. Phys.* **49**(8-9-10) 729